Calculus A Final Review Worksheet #2 Name: Key

1. $\lim_{x\to\infty}\frac{x^3}{e^{2x}} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{3x^2}{2e^{2x}} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{\cancel{3}\,\cancel{6}x}{\cancel{2}\,\cancel{4}e^{2x}} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{3}{4e^{2x}}$

$= \frac{3}{\infty}$

$= \boxed{0}$

2. Find the slope of the tangent line for $f(x)=\ln\left(x^2e^{-x}\right)$ at the point where $x=2$

$f'(x) = \frac{1}{x^2e^{-x}}\left(x^2(-e^{-x}) + 2xe^{-x}\right)$

$= \frac{xe^{-x}(-x+2)}{x^2e^{-x}}$

$= \frac{2-x}{x} \rightarrow \boxed{f'(2)=0}$

OR $f(x) = 2\ln x - x\ln e$

$f(x) = 2\ln x - x(1)$

$f'(x) = \frac{2}{x} - 1$

$f'(x) = \frac{2}{2} - 1$

$\boxed{f'(x) = 0}$

3. Find the derivative for $f(x) = -\csc\left(\sqrt[3]{x}\right)$

$f'(x) = -\left(-\csc\sqrt[3]{x}\cot\sqrt[3]{x}\right)\left(\frac{1}{3}x^{-2/3}\right)$

$= \frac{\csc\sqrt[3]{x}\cot\sqrt[3]{x}}{3\sqrt[3]{x^2}}$

4. Find $\frac{dy}{dx}$ if $y=\cot(x+y)$

$\frac{dy}{dx} = \left(-\csc^2(x+y)\right)\left(1+\frac{dy}{dx}\right)$

$\frac{dy}{dx} = -\csc^2(x+y) - \frac{dy}{dx}\csc^2(x+y)$

$\frac{dy}{dx}\left(1+\csc^2(x+y)\right) = -\csc^2(x+y)$

$\boxed{\frac{dy}{dx} = -\frac{\csc^2(x+y)}{1+\csc^2(x+y)}}$

5. Find $y'$ if $y=\arcsin\sqrt{x}$ $\longrightarrow$ $u=\sqrt{x}$ ; $u' = \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{1-x}}$

$\boxed{y' = \frac{1}{2\sqrt{x-x^2}}}$

6. Find the extrema on the interval $[0,\pi]$ if $y=\sin x - x$

closed interval, so check endpts!

$y' = \cos x - 1$

$0 = \cos x - 1$

$1 = \cos x$

$x = 0$ ← critical # is also an endpt.

@ $x=0$ ; $y = \sin 0 - 0$

$\boxed{(0,0) \text{ Absolute Max}}$

@ $x=\pi$ ; $y = \sin\pi - \pi$

$\boxed{(\pi,-\pi) \text{ Absolute Min}}$

7. Find $y'$ if $y = e^{-\cos(x^2)}$

$\ln y = (-\cos x^2)\ln e$ ← $\ln e = 1$

$\ln y = -\cos(x^2)$

$\frac{1}{y}\frac{dy}{dx} = (\sin x^2)(2x)$

$\frac{dy}{dx} = 2x\sin x^2 \cdot e^{-\cos x^2}$

$\frac{dy}{dx} = \left(2xe^{-\cos x^2}\right)\left(\sin x^2\right)$

★ parametric equations → $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ ← derivative of y in terms of t
← derivative of x in terms of t

8. Find $\frac{dy}{dx}$ if $x=\sqrt[3]{t}$ and $y=(t+1)^2$

$$\frac{dy}{dx} = \frac{2(t+1)(1)}{\frac{1}{3}t^{-2/3}} = \frac{2t+2}{\frac{1}{3t^{2/3}}}$$

$$=(2t+2)(3t^{2/3})$$

$$\boxed{=6t^{5/3}+6t^{2/3} \quad \text{or} \quad 6t^{2/3}(t+1)}$$

9. Find $\frac{d^2y}{dx^2}$ if $x=\frac{1}{3}t^3$ and $y=2t-1$

$$\frac{dy}{dx} = \frac{2}{t^2}$$

$$\frac{dy}{dx} = 2t^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$$

← Derivative of your first derivative

← the bottom of your first derivative before you simplified.

$$\frac{d^2y}{dx^2} = \frac{-4t^{-3}}{t^2}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{4}{t^5}}$$

10. Find the equation of the tangent line for the curve $x=3t^2$, $y=(t-1)^2$ at the point where $t=1$

slope → $\frac{dy}{dx} = \frac{2(t-1)(1)}{6t}$

$$= \frac{t-1}{3t}$$

$x=3(1)^2=3$

$y=(1-1)^2=0$

$(3,0)$

@ $t=1$ → slope $= \frac{1-1}{3(1)} = 0$ horizontal line $\boxed{y=0}$

11. Find the slope of the tangent line for the curve $r=\cos\theta-1$ at the point where $\theta=\frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$r' = -\sin\theta$

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$\frac{dy}{dx} = \frac{(\cos\theta-1)\cos\theta + (-\sin\theta)\sin\theta}{-(\cos\theta-1)\sin\theta + (-\sin\theta)\cos\theta}$$

→ @ $\theta=\frac{\pi}{3}$, slope $= \frac{\left(\frac{1}{2}-1\right)\left(\frac{1}{2}\right) + -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}{-\left(\frac{1}{2}-1\right)\frac{\sqrt{3}}{2} + -\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}$

$$= \frac{-\frac{1}{4}-\frac{3}{4}}{\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}} = \frac{-1}{0}$$

Just wants slope @ $\theta=\frac{\pi}{3}$,
so no need to simplify

$\boxed{\text{slope is undefined}}$

12. Find $r'(t)$ if $r(t)=e^{2t}i+3t^2j$

$$r'(t) = 2e^{2t}i + 6tj$$